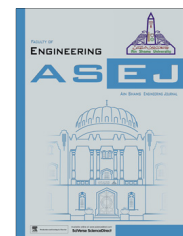




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A novel reliability evaluation method for large engineering systems



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Abstract A novel reliability evaluation method for large nonlinear engineering systems excited by dynamic loading applied in time domain is presented. For this class of problems, the performance functions are expected to be function of time and implicit in nature. Available first- or second-order reliability method (FORM/SORM) will be challenging to estimate reliability of such systems. Because of its inefficiency, the classical Monte Carlo simulation (MCS) method also cannot be used for large nonlinear dynamic systems. In the proposed approach, only tens instead of hundreds or thousands of deterministic evaluations at intelligently selected points are used to extract the reliability information. A hybrid approach, consisting of the stochastic finite element method (SFEM) developed by the author and his research team using FORM, response surface method (RSM), an interpolation scheme, and advanced factorial schemes, is proposed. The method is clarified with the help of several numerical examples.

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1. Introduction

The risk management has become an essential responsibility of the engineering profession since risk in engineering design cannot completely be eliminated. The issue has attracted added significance since the basic design philosophy has changed from human safety to structural safety. The enormous amount

of damage caused to infrastructures during recent earthquakes in China, Chile, Haiti, India, Iran, Japan, U.S., and other parts of the world prompted this change. The word “structure” is used here in a generic sense. It represents real engineered systems that can be represented by finite elements; including structures in a nuclear power plant, multi-story buildings, bridges, on-shore and offshore structures, soil-pile interaction problems in offshore mooring systems, aero-space structures, etc.

Appropriate risk management requires an acceptable reliability method considering all major loads and load combinations that may act on the structures during their lifetime and analyzing their nonlinear behavior as realistically as possible just before failure satisfying the underlying physics. Unfortunately, a robust reliability evaluation technique for large structural systems may not be available at this time.

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Nomenclature

A^b, A^c	gross area of beam and column; respectively.	P_f	the probability of failure
$b_0, b_i, b_{ii},$ and b_{ij}	unknown coefficients of a polynomial to be determined	r	the pile radius
CCD, SD	central composite design and saturated design	SD, CCD	saturated and central composite design
\mathbf{D}	the global displacement vector	\mathbf{Q}	the transformation matrix
E, E_c	modulus of elasticity of steel and concrete, respectively	RS	response surface
EDSD	Explicit Design Space Decomposition	RSM	response surface method
FEM	finite element method	SFEM	a FEM- and FORM-based reliability analysis method
FORM	First Order Reliability Method	SORM	Second Order Reliability Method
f'_c	the compressive strength of concrete	SVM	Support Vector Machines
f_r	the allowable tensile strength of concrete	t	the pile thickness
F_y	the yield strength	\mathbf{x}	the basic random variables
$g(\mathbf{X})$	explicit expression of the limit state function	$\mathbf{x}_{C_1}, \mathbf{x}_{C_2}$	first and second center point
$\hat{g}(\mathbf{X})$	response surface function	\mathbf{x}_{D_1}	the coordinates of the checking point
g_e	magnification factor for the amplitude of actual seismic acceleration	$X_i (i = 1, 2, \dots, k)$	the i th random variable
H	the horizontal load	X_i^C	the coordinates of the center point, i
h_i	a chosen factor that defines the experimental/sample region	x, u, s	set of basic random variables, displacements and load
HDMR	High Dimensional Model Representation	\mathbf{y}	transformed random variables in standard normal space
HORM	high-order response surface method	$y_{max}(x)$	maximum lateral displacement
I	the pile moment of inertia	$\nabla g(\mathbf{y})$	the response gradient
I^b, I^c	beam and column moment of inertia	Z_x^b, Z_x^c	plastic modulus of beam and column; respectively
$\mathbf{J}_{i,j}$'s	the Jacobians of transformation ($\mathbf{J}_{y,x} = \partial \mathbf{y} / \partial \mathbf{x}$, $\mathbf{J}_{s,D} = \partial \mathbf{s} / \partial \mathbf{D}$, $\mathbf{J}_{D,x} = \partial \mathbf{D} / \partial \mathbf{x}$ and $\mathbf{J}_{s,x} = \partial \mathbf{s} / \partial \mathbf{x}$)	ξ	the damping ratio
k	the number of random variables	ε	pre-selected convergence criterion
L_0	the pile length above the mud line	$\alpha(X_i)$	sensitivity indexes of the variable X_i
m	total number of most sensitive random variables	β	β -index = Reliability index
MCS	Monte Carlo simulation	$\partial g / \partial \mathbf{s}, \partial g / \partial \mathbf{u}$, and $\partial g / \partial \mathbf{x}$	three partial derivatives of basic random variables, displacements and load effects
MDS	the mooring dolphin structures	δ_{allow}	the allowable drift
MPFP	the most probable failure point	Φ	the standard normal cumulative distribution function
p	the numbers of coefficients necessary to define a polynomial	ε	pre-selected convergence criterion
		σ_{x_i}	The standard deviation of a random variable X_i

Risk analysis methods currently available were developed for simple systems with numerous assumptions which cannot be satisfied for large systems [1]. At present, the most commonly used reliability methods are the first-order and/or second-order reliability methods (FORM/SORM) [2]. They require that the performance functions to be available in explicit forms. The algorithm is iterative in nature and the gradients of performance or limit state functions are required to estimate the coordinates of the most probable failure point (MPFP), the corresponding reliability index, and the failure probability. For large complicated structural systems, the limit states are expected to be implicit. In response to questions on future research directions in the risk evaluation techniques, Rackwitz [3] commented that the use of FORM/SORM complemented by Monte Carlo simulations (MCS) would be the next frontier. As will be discussed later, several recent studies reflected this idea and simulation has become an integral part of reliability evaluation studies.

When the limit state function is implicit, one of the options is to use MCS for the reliability evaluation. The authors observed that one deterministic nonlinear analysis of large structures may take over 10 h of computer time. If one has

to use very small, say only 10^3 simulations, it may take 10^4 h or over 1.14 years of uninterrupted running of a computer. This simple example clearly indicates that simulation, even with sophisticated variance reduction schemes [2], including importance sampling [4], directional sampling [5], subset simulation [6], etc., may not be attractive alternatives for low probability events; a new reliability method must be developed for large structural systems. A new method, an alternative to the classical MCS is proposed for this purpose in this paper. In this approach, only tens instead of thousands of deterministic evaluations at intelligently selected points are used to extract the reliability information.

The probability of failure implies that it needs to be estimated just before failure developing various sources of nonlinearities. Also, the most sophisticated analysis of such structure requires that the load should be applied as realistically as practicable. The finite element method (FEM) is commonly used to study nonlinear behavior for this class of problems. The discussion clearly indicates that a FEM-based general purpose reliability evaluation method is necessary for the reliability evaluation. This will also satisfy the deterministic community since the procedure will be within their areas of expertise and

the classical random vibration-based approaches may not provide the required reliability information. Acceptance of any new reliability evaluation method by the deterministic community is essential since they are in the majority and generally make the final decision, in most cases.

Structural elements (beams, columns, connections, etc.) are generally designed first for strength according to design guidelines given in codes satisfying some underlying reliability requirements, although unknown to most designers. However, the overall system reliability in strength considering reliabilities of all the elements remains unknown. It is not addressed in codes since it is difficult to estimate and can only be assessed based on numerous idealistic assumptions (brittle or ductile behavior, dependency of failure of elements, consideration of nonlinear limit states, etc.) [2]. Moreover, the design satisfying strength requirements may not satisfy the global performance or serviceability requirement, e.g., excessive lateral deflection, a major cause of structural failure for seismic loading. Serviceability requirements also may control the seismic design in most cases and should not be ignored.

2. Related works and available reliability methods

Eliminating the basic MCS as a realistic alternative to estimate reliability of low probability events for large structural systems, the available computational approaches can be broadly divided into two categories: (i) the sensitivity-based stochastic finite element method (SFEM) and (ii) the response surface method (RSM). The estimation of the probability of failure implies that the risk needs to be estimated just before failure. Different sources of nonlinearity expected to develop just before failure can be routinely incorporated in the deterministic FEM-based formulations. This led to the development of sensitivity-based SFEM formulation [7]. The other alternative is RSM [8]. The primary purpose of applying RSM in reliability analysis is to approximate the original complex and implicit limit state function using a simple and explicit polynomial [9–11]. Three basic weaknesses of RSM that need to be addressed before applying it for the structural reliability evaluation are as follows: (1) it cannot incorporate distribution information of random variables even when it is available, (2) if the response surface (RS) is not generated in the failure region, it may not be directly applicable or robust, and (3) for large systems, it may not give the optimal sampling points. Thus, a basic RSM-based reliability method may not be acceptable.

At present, second order polynomial without and with cross terms is generally used to generate RSs. Recently, Li et al. [12] proposed high-order response surface method (HORSM). The method employs Hermite polynomials and the one-dimensional Gaussian points as sampling points to determine the highest power of each variable. In recent past, several methods with the general objective of approximately developing multivariate expressions for response surface for mechanical engineering applications were proposed. One such method is High Dimensional Model Representation (HDMR) [13–21]. It is also referred to as “Decomposition method”, “Univariate approximation”, “Bivariate approximation”, “S-variate approximation”, etc. HDMR captures the high-dimensional relationships between sets of input and output model variables in such a way that the component functions of the approximation are ordered starting from a constant

and adding terms such as first order, second order, and so on. The concept appears to be reasonable if higher-order variable correlations are weak, allowing the physical model to be captured by the first few lower-order terms.

Another major work is known as the Explicit Design Space Decomposition (EDSD). It can be used when responses can be classified into two classes, e.g., safe and unsafe. The classification is performed using explicitly defined boundaries in space. A machine learning technique known as Support Vector Machines (SVM) [22–25] is used to construct the boundaries separating distinct classes. The failure regions corresponding to different modes of failure are represented with a single SVM boundary, which is refined through adaptive sampling.

The HORSM, HDMR and EDSD-SVM approaches use MCS to estimate the underlying reliability at the last stage. They may not be suitable for engineering applications where several sources of nonlinearities must be explicitly analytically incorporated in the formulation satisfying the underlying physics. A new method needs to be developed.

3. Proposed method

The proposed reliability evaluation method for large structural systems is developed in three stages. In the first stage, the two weaknesses of RSM, i.e., the consideration of distributional information of the random variables present in the formulation and identification of the location of the failure region, are addressed by integrating it with FORM/SORM. This approach will lead to a hybrid approach consisting of SFEM, FORM/SORM, and RSM. Since the integration produces an iterative approach, in the second stage, the efficiency of the method is improved by using several iterative approaches to generate the RS. However, this improvement in efficiency may not be adequate for large structural systems; the formulation needs to be improved further. In the third stage, several improved factorial schemes are used to obtain the required response surfaces requiring fewer sampling points. All these stages are briefly discussed below.

3.1. Stochastic finite element method

A FEM- and FORM-based reliability analysis method is under development by the first author and his research team. It is known as the stochastic FEM or SFEM method [7,26]. In this approach, structures are represented by two- or three-dimensional frame elements and the FEM is used to solve the deterministic nonlinear governing equation. Different sources of nonlinearity can be incorporated without losing the basic simplicity. It is very accurate and adds efficiency in the deterministic analysis of nonlinear frame structures.

Risk is always estimated with respect to a performance or limit state function. Without losing any generality, the limit state function can be expressed as $g(\mathbf{x}, \mathbf{u}, \mathbf{s}) = 0$, where \mathbf{x} is a set of basic random variables (e.g., loads, material properties and structural geometry), \mathbf{u} is the set of displacements and \mathbf{s} is the set of load effects (except the displacements, such as internal forces). For reliability computation, it is convenient to transform \mathbf{x} into the standard normal space $\mathbf{y} = \mathbf{y}(\mathbf{x})$ such that the elements of \mathbf{y} are statistically independent and have a standard normal distribution. An iterative algorithm can be used to locate the design point (the most likely failure point)

on the limit state function using the first-order approximation. The structural response and the response gradient vectors are calculated using finite element models at each iteration. The following iteration scheme can be used for finding the coordinates of the design point:

$$\mathbf{y}_{i+1} = \left[\mathbf{y}'_i \alpha_i + \frac{g(\mathbf{y}_i)}{|\nabla g(\mathbf{y}_i)|} \right] \alpha_i \quad (1)$$

where

$$\nabla g(\mathbf{y}) = \left[\frac{\partial g(\mathbf{y})}{\partial y_1}, \dots, \frac{\partial g(\mathbf{y})}{\partial y_n} \right]^T \text{ and } \alpha_i = -\frac{\nabla g(\mathbf{y}_i)}{|\nabla g(\mathbf{y}_i)|} \quad (2)$$

To implement the algorithm, the gradient $\nabla g(\mathbf{y})$ of the limit state function in the standard normal space can be derived as [7]

$$\nabla g(\mathbf{y}) = \left[\frac{\partial g(\mathbf{y})}{\partial \mathbf{s}} \mathbf{J}_{s,x} + \left(\mathbf{Q} \frac{\partial g(\mathbf{y})}{\partial \mathbf{u}} + \frac{\partial g(\mathbf{y})}{\partial \mathbf{s}} \mathbf{J}_{s,D} \right) \mathbf{J}_{D,x} + \frac{\partial g(\mathbf{y})}{\partial \mathbf{x}} \right] \mathbf{J}_{y,x}^{-1} \quad (3)$$

where $\mathbf{J}_{i,j}$'s are the Jacobians matrixes ($\mathbf{J}_{y,x} = \partial \mathbf{y} / \partial \mathbf{x}$, $\mathbf{J}_{s,D} = \partial \mathbf{s} / \partial \mathbf{D}$, $\mathbf{J}_{D,x} = \partial \mathbf{D} / \partial \mathbf{x}$ and $\mathbf{J}_{s,x} = \partial \mathbf{s} / \partial \mathbf{x}$); \mathbf{x} is the basic random variables; y_i 's are statistically independent random variables in the standard normal space, \mathbf{Q} is the transformation matrix, \mathbf{D} is the global displacement vector. The evaluation of the quantities in Eq. (3) will depend on the problem under consideration (linear or nonlinear, two- or three-dimensional, etc.) and the performance functions used. The essential numerical aspect of SFEM is the evaluation of three partial derivatives, $\partial g / \partial \mathbf{s}$, $\partial g / \partial \mathbf{u}$, and $\partial g / \partial \mathbf{x}$, and four Jacobians, $\mathbf{J}_{s,x}$, $\mathbf{J}_{s,D}$, $\mathbf{J}_{D,x}$, and $\mathbf{J}_{y,x}$. They can be evaluated by procedures suggested in [7]. Once the coordinates of the design point \mathbf{y}^* are evaluated with a preselected convergence criterion, the reliability index β can be evaluated as follows:

$$\beta = \sqrt{(\mathbf{y}^*)^T (\mathbf{y}^*)} \quad (4)$$

The evaluation of Eq. (4) will depend on the problem under consideration and the limit state functions used. The probability of failure, P_f can be calculated as

$$P_f = \Phi(-\beta) = 1.0 - \Phi(\beta) \quad (5)$$

where Φ is the standard normal cumulative distribution function. Eq. (5) can be considered as a notational failure probability. When the reliability index is larger, the probability of failure will be smaller [39].

3.2. Proposed method – stage 1 – integration of SFEM and RSM

As mentioned earlier, the SFEM concept briefly discussed in the previous section may not be able to estimate reliability of large complicated structural systems under complicated loadings since the limit state functions will be implicit in nature. The research team concluded that it should be integrated with RSM.

3.3. Response surface method

As discussed earlier, three basic weaknesses of RSM need to be addressed before integrating it with SFEM. The two major weaknesses of RSM, i.e., the consideration of distributional

information of the random variables present in the formulation and identification of the location of the failure region, are addressed by integrating it with FORM. This approach will lead to a hybrid approach consisting of SFEM, FORM, and RSM.

In implementing any RSM-based scheme, three issues that need consideration are as follows: (1) the degree of polynomial to be used to generate the response surface, (2) the location of center points, and (3) experimental sampling points. Considering the fact that higher order polynomial may result in ill-conditional system of equations for unknown coefficients and exhibit irregular behavior outside of the domain of samples, their utilization in generating RSM has received relatively little attention [27,28]. For complicated problems considered by the research team, second-order polynomial, without and with cross terms, is considered to be appropriate. They can be expressed as follows:

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^k b_i X_i + \sum_{i=1}^k b_{ii} X_i^2 \quad (6)$$

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^k b_i X_i + \sum_{i=1}^k b_{ii} X_i^2 + \sum_{i=1}^{k-1} \sum_{j>i}^k b_{ij} X_i X_j \quad (7)$$

where X_i ($i = 1, 2, \dots, k$) is the i th random variable, k is the number of random variables in the formulation, ($j = 1, 2, \dots, k$) and b_0 , b_i , b_{ii} , and b_{ij} are unknown coefficients to be determined. The numbers of coefficient necessary to define Eqs. (6) and (7) are $p = 2k + 1$ and $p = (k + 1)(k + 2)/2$, respectively. The coefficients can be fully defined by estimating deterministic responses at intelligently selected data points called experimental sampling points. The concept behind a sampling scheme can be expressed as follows:

$$X_i = X_i^C \pm h_i \sigma_{x_i} x_i \quad (8)$$

where X_i^C and σ_{x_i} are the coordinates of the center point and the standard deviation of a random variable X_i , respectively; h_i is an arbitrary factor that defines the experimental region.

Sampling points are selected around the center point. The selection of the center point and experimental sampling points around it are crucial factors in establishing the efficiency and accuracy of the proposed iterative method. In the context of iterative scheme of FORM, the initial center point \mathbf{x}_{C1} is selected to be the mean values of the random variable X_i 's. Then, using the responses obtained from the deterministic FEM evaluations for all the experimental sampling points around the center point, the response surface $\hat{g}_1(\mathbf{X})$ can be generated explicitly in terms of the random variables \mathbf{X} . Once a closed form expression for the limit state function is obtained, the coordinates of the checking point \mathbf{x}_{D1} can be estimated using FORM, using all the statistical information on the X_i 's, eliminating one major deficiency of RSM. The response can be evaluated again at the checking point \mathbf{x}_{D1} , and a new center point \mathbf{x}_{C2} can be selected using linear interpolation from the center point \mathbf{x}_{C1} to \mathbf{x}_{D1} such that $g(\mathbf{X}) = 0$; i.e.

$$\mathbf{x}_{C2} = \mathbf{x}_{C1} + (\mathbf{x}_{D1} - \mathbf{x}_{C1}) \frac{g(\mathbf{x}_{C1})}{g(\mathbf{x}_{C1}) - g(\mathbf{x}_{D1})} : \text{if } g(\mathbf{x}_{D1}) \geq g(\mathbf{x}_{C1}) \quad (9)$$

$$\mathbf{x}_{C2} = \mathbf{x}_{D1} + (\mathbf{x}_{C1} - \mathbf{x}_{D1}) \frac{g(\mathbf{x}_{D1})}{g(\mathbf{x}_{D1}) - g(\mathbf{x}_{C1})} : \text{if } g(\mathbf{x}_{D1}) < g(\mathbf{x}_{C1}) \quad (10)$$

A new center point \mathbf{x}_{C_2} then can be used to develop an explicit performance function for the next iteration. This iteration scheme can be repeated until a pre-selected convergence criterion of $(\mathbf{x}_{C_{i+1}} - \mathbf{x}_{C_i})/\mathbf{x}_{C_i} \leq \varepsilon$ is satisfied. ε is considered to be $|0.05|$ in this study. It is assumed that this accuracy is sufficient for the structural reliability analysis and was assumed previously in the literature [1]. In the final iteration, the information on the most recent center point is used to formulate the final RS. FORM is then used to calculate the reliability index and the corresponding coordinates of the most probable failure point.

3.4. Proposed method – stage 2 – efficient schemes for sampling points

To select experimental sampling points around the center point, saturated design (SD) and central composite design (CCD) are very promising schemes. SD is less accurate but more efficient since it requires only as many sampling points as the total number of unknown coefficients to define the response surface. CCD is more accurate but less efficient since a regression analysis needs to be carried out to evaluate the unknown coefficients for the RS.

To illustrate the computational effort required for the reliability evaluation of large structural system, suppose the total number of random variables present in the formulation is, $k = 20$. The total number of coefficients necessary to define Eq. (6) will be $2 \times 20 + 1 = 41$ and to define Eq. (7) will be $(20 + 1)(20 + 2)/2 = 231$. Since the proposed algorithm is iterative and the basic SD and CCD require different amount of computational effort, considering efficiency without compromising accuracy, several schemes can be followed. Among numerous schemes considered by the research team, one basic and two promising schemes are as follows:

Scheme 0 – SD using 2nd order polynomial without the cross terms throughout all the iterations.

Scheme 1 – Eq. (6) and SD for intermediate iterations and Eq. (7) and full SD for the final iteration.

Scheme 2 – Eq. (6) and SD for intermediate iterations and Eq. (7) and CCD for the final iteration.

The basic concepts behind these schemes are shown in Fig. 1. Considering the above three schemes, the total number of FE analyses required to generate the necessary response surface is $2k + 1$, $(k + 1)(k + 2)/2$ and $2^k + 2k + 1$, respectively, where k is the total number of random variables in the formulation. To demonstrate the computational effort needed to implement the three schemes, for $k = 20$, the total number of required FE analyses will be 41, 231, and 1,048,617, respectively, for the above three schemes. For large real structural systems, the total number of random variables could be much larger and the above three schemes may not be applicable or practical. The accuracy of Scheme 0 is an open question and is not considered further.

3.5. Proposed method – stage 3 – advanced schemes

The efficiency of Scheme 1 can be improved further by using advanced factorial scheme, as discussed below. It is denoted as Scheme M1.

Scheme M1: To improve the efficiency of Scheme 1, the cross terms (edge points), $k(k-1)$, are suggested to be added only for the most important variables in the last iteration. Since the proposed algorithm is an integral part of FORM/SORM, all the random variables in the formulation can be arranged in descending order of their sensitivity indexes $\alpha(X_i)$, i.e. $\alpha(X_1) > \alpha(X_2) > \alpha(X_3) \dots > \alpha(X_k)$. The sensitivity of a variable X , $\alpha(X)$ is the directional cosines of the unit normal vector at the design point. In the last iteration, the cross terms are added only for the most sensitive random variables, m and the corresponding reliability index is calculated. The total number of FEM analyses required for Scheme 1 and M1 is $(k + 1)(k + 2)/2$ and $2k + 1 + m(2k - m - 1)/2$, respectively. For an example, suppose for a large structural system, $k = 20$ and $m = 3$. The total number of required FEM analyses will be 231 and 95, respectively, for the two schemes. The improvement significantly improves the efficiency.

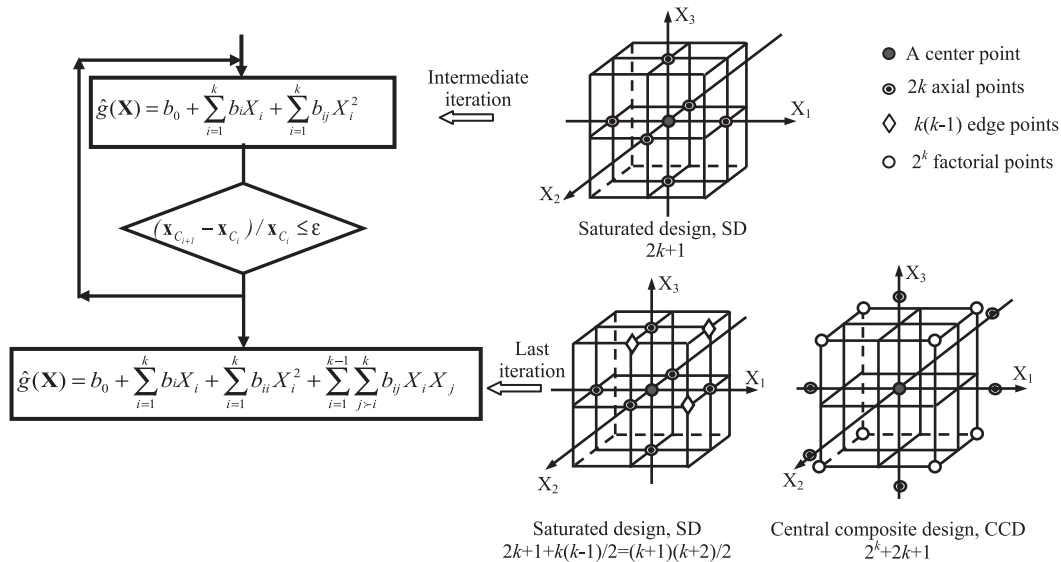


Figure 1 Scheme 0, scheme 1 and scheme 2 (coded variable space $k = 3$).

4. Implementation of the proposed method

The authors suggest the following two steps to implement the proposed procedure.

Step 1: Conduct a preliminary reliability analysis and reduce the total number of random variables using the sensitivity analysis. To generate necessary information on sensitivity indexes, initially the authors used the first-order instead of second-order RS and SD scheme to generate it. Then, less sensitive random variables are considered to be deterministic at their mean value. The implications of this step will be elaborated in the second example. This step is preliminary and optional.

Step 2: Complete the reliability evaluation of the system using second-order RS and different schemes as proposed in this study. This step is essential and is shown in the flowchart in Fig. 1.

1. Initially, scheme 0, i.e., SD with second-order polynomial without cross terms can be used to generate the necessary RS using several deterministic modeling assumptions to capture the realistic behavior.
2. Then, the corresponding reliability index or the probability of failure, P_f , can be estimated using Scheme 1 or Scheme 2 and FORM.
3. Subsequently, M1 scheme is considered to estimate P_f .

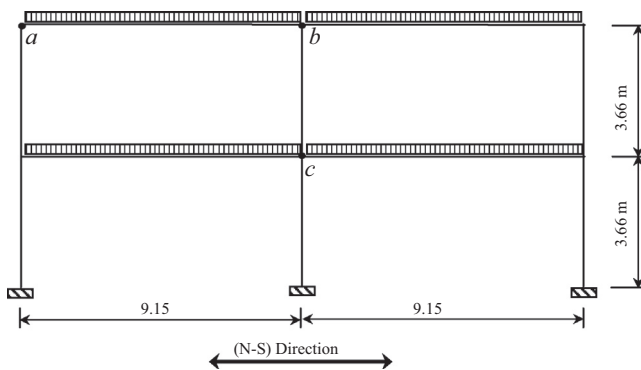


Figure 2 A Frame without Shear Walls.

5. Examples

The capabilities, robustness, and efficiency of the methods are demonstrated with two examples. In this example 1, an in-house FE code and FORM are utilized with Schemes 1 and 2, and Eqs. (1)–(5), while in example 2, a commercial FE program (COSMOS) and SORM are used with all schemes and the advanced scheme M1, using Eqs. (6)–(10). All the results are verified using the Monte Carlo simulation method. It should be mentioned that, comparison between FORM and SORM was not the objective of the present paper. The objective was how to improve the efficiency (minimize the number of FE calls or FE simulations) without compromising the accuracy. However, the result using FORM is also recorded.

5.1. Example 1 – reliability analysis of a steel frame without and with RC shear walls

A two-story three-bay steel frame, as shown in Fig. 2, is considered [29,30]. Section sizes of beams and columns, using A36 steel, are given in Table 1. The fundamental period of the frame is found to be 0.778 s. The gravity load acting on each of the two floors is 35.04 kN/m. A building is supposed to consist of several such frames. The frame was excited for 5.12 s by the El Centro Earthquake (N-S component) of 1940 as shown in Fig. 3. To address the uncertainty in the amplitude of seismic excitation, a parameter g_e is introduced. The damping coefficient ξ is expressed as a percent of the critical damping. The statistical information of g_e and ξ is given in Table 1.

The bare frame without any lateral reinforcement is found to be very weak when excited by the seismic loading. Its strength in the lateral direction needs to be improved. The serviceability limit state of the overall lateral displacement at the top of the frame is specifically considered in this example. The serviceability performance function is expressed as follows:

$$g(\mathbf{x}) = \delta_{allow} - y_{max}(\mathbf{x}) = \delta_{allow} - \hat{g}(\mathbf{x}) \quad (11)$$

where δ_{allow} is the prescribed horizontal displacement and $y_{max}(x)$ is the corresponding overall lateral displacement. For the example, the prescribed horizontal displacement is considered

Table 1 Basic random variables of frame and shear wall-Example 1.

Parts	Variables	Nominal	Mean/Nominal	C.O.V	Distribution	Comment
Frame	E (Mpa)	2.0×10^5	1.0	0.06	Lognormal	Beam W18 \times 60
	F_y (Mpa)	248.2	1.05	0.1	Lognormal	
	A^b (cm ²)	113.6	1.0	0.05	Lognormal	
	I^b (cm ⁴)	40,957	1.0	0.05	Lognormal	
	Z_x^b (cm ³)	2015	1.0	0.05	Lognormal	Column W12 \times 40
	A^c (cm ²)	76.1	1.0	0.05	Lognormal	
	I^c (cm ⁴)	12,903	1.0	0.05	Lognormal	
	Z_x^c (cm ³)	942.3	1.0	0.05	Lognormal	
Wall	E_c (Mpa)	2.14×10^4	1.0	0.18	Lognormal	
	N	0.17	1.0	0.10	Lognormal	
Dynamic loading parameters	ξ	0.02	1.0	0.15	Lognormal	Without shear walls
	g_e	1.0	1.0	0.2	Type I	With shear walls
	ξ	0.05	1.0	0.15	Lognormal	
	g_e	1.0	1.0	0.2	Type I	

b: Beam; c: Column; ξ : Damping ratio; g_e : Magnification factor for the amplitude of actual seismic acceleration.

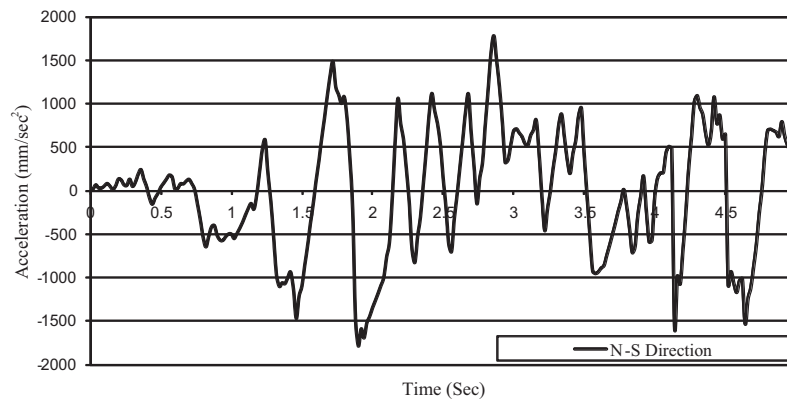


Figure 3 El Centro Earthquake time history for 5.12 s (N-S component).

Table 2 Probability of failure of the frame without and with shear walls-example 1.

	Proposed method			MCS (100,000 cycles)	
	Scheme	p_f	CPU (s)	p_f	CPU (s)
Without shear wall	Scheme 1	0.9999	134	1.0	98,459
With shear walls	Scheme 1	0.0057	202	0.0049	117,832
	Scheme 2	0.0094	295		

CPU time is for SGI Origin Supercomputer, 2000.

not to exceed $h/400$, where h is the height of the frame. Thus, δ_{allow} is 1.83 cm for this example.

Considering all the random variables identified in Table 1 and the serviceability performance function represented by Eq. (11), the probability of failure of the frame due to horizontal displacement at Node a is calculated. The results are summarized in Table 2. Only scheme 1 is used for the reliability evaluation of the steel frame. (As the probability of failure is close to 1.) The probability of failure is very close to 1.0 indicating that the bare frame is unable to carry the seismic load. It needs reinforcement in the horizontal direction. The probability of failure according to 10^5 cycles MCS is very similar to the proposed method. However, the CPU time required for the proposed algorithm (134 s) is about 0.136% of that of MCS (98,459 s), indicating the proposed algorithm is very efficient. The CPU times given here are for a supercomputer SGI Origin 2000.

Since the frame cannot carry the seismic load, it was reinforced with reinforced concrete (RC) shear walls as shown in Fig. 4. Two additional random variables related to the RC shear walls, E_c and ν , need to be considered. Their statistical properties are given in Table 1. The formation of cracks in RC shear walls is also considered. After the tensile stress of shear walls exceeds the prescribed tensile stress of concrete, the degradation of the stiffness is assumed to be reduced to 40% of the original stiffness [31]. The shear walls are assumed to develop cracks when the tensile stress in concrete exceeds the rupture strength. The rupture strength of concrete, f_r , according to the American Concrete Institute [32] is assumed to be $f_r = 7.5 \times \sqrt{f'_c}$, where f'_c is the compressive strength of concrete. It is considered to be 20.68 Mpa in this study.

The fundamental period of the reinforced frame is found to be 0.313 s. The frame is again excited by the same El Centro

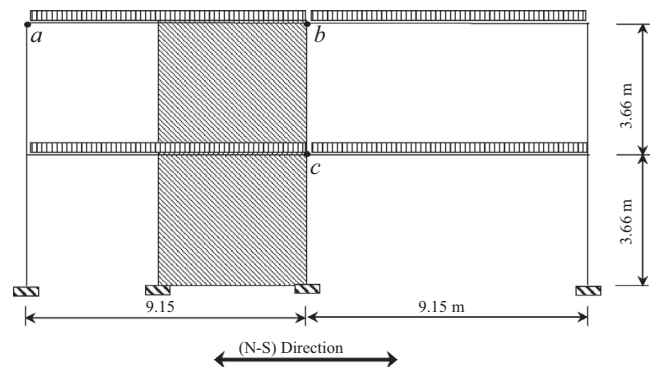


Figure 4 The frame reinforced with RC shear walls.

Earthquake time history, as before. The probability of failure of the combined system is then calculated using the proposed method. The results are summarized in Table 2, for Schemes 1 and 2. As expected, the presence of RC shear walls significantly improves the serviceability behavior of the steel frame. Again, the results obtained by the proposed method were verified by using 10^5 MCS simulation cycles. The results clearly indicate that both Schemes 1 and 2 can accurately estimate the probability of failure of the combined system. However, considering the CPU time, Scheme 1 is more efficient.

It should be clear to the reader that the aim of the above example is not to compare the reliability of a frame without and with shear wall. However, the objective is to verify the in-house (written by the research team) FE- and FORM-based code. Large example will be prohibited in the verification using Monte Carlo simulation. Part of the above example was published before [30] under the subject of shear wall-frame

structures; however, it is considered here for the sake of completeness with the new improvement in response surface scheme (scheme M1 in the next example). Moreover, the wall-frame structure -under earthquake load- is considered here under the title of reliability of complex engineering systems.

5.2. Example 2 – reliability analysis of mooring dolphin structures

Analysis and design of offshore mooring dolphin structures (MDS) are very challenging due to their complicated structural arrangements, different materials needed to build them, different mathematical models and material behavior used to study their response, different loading environments they are exposed to, etc. In such a system, piles are driven in soil and they become an integral part of the offshore foundation structures to carry the mooring loads. A typical layout/pattern of MDS is shown in Fig. 5.

The basic elements common to mooring system include mooring and breasting structures, mooring lines, deck fittings, separators, access trestles and catwalks [33]. The mooring loads are generally caused by wind and currents producing longitudinal and/or lateral forces on the ship or the vessel. The longitudinal forces are generally conservatively assumed to be resisted by the spring lines and the lateral loads are resisted by breast dolphins. The afterward lateral force is resisted by mooring dolphin.

External events such as collisions with other ships, sea state, wind, ice, and other weather-related incidences will also dictate the loading conditions a MDS will be subjected to during its operation. Considering the uniqueness of the problem, uncertainty associated with important variables in the formulation is quantified first. After that, it is used to quantify the relative importance or influence (sensitivity index) of different design variables on the estimated risk.

Once the sensitivity indexes of the design random variables are known, the information can be used to control or manage them in the most effective way; producing an economical design.

To study the behavior of complicated structural arrangements of a typical MDS system consisting of a steel pile partially embedded in soil deposit below the mud line, its top part exposed to water is expected to be very complicated. The soil around the pile is expected to significantly influence the overall behavior of the steel pile-soil system. The MDS is

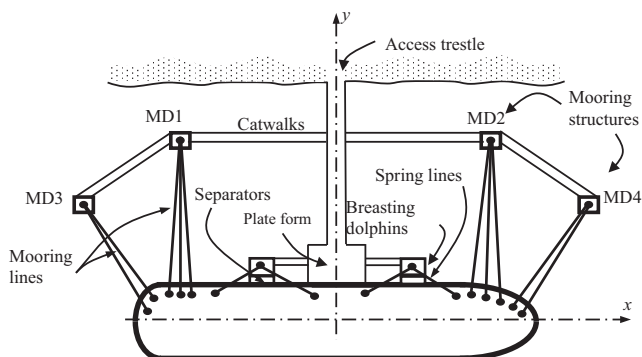


Figure 5 A typical mooring system.

represented by finite elements (FEs). The soil is often represented by 8-node solid elements while the pile is represented by 3D beam element, as shown in Fig. 6.

In this example, a large diameter open steel pipe pile is used as a mooring dolphin structure. The pile has radius $r = 0.95$ m, thickness $t = 2.8$ cm and an unsupported length $L_o = 17.50$ m. The mooring design load is $H = 1500$ kN and the modulus of elasticity is $E = 2.01 \times 10^8$ kN/m². The information of random variables is summarized in Table 3. The mooring force, H , is assumed to have the same statistical properties as the wind load; Gumbel/EV-I distribution with a COV of 0.37.

For the sake of simplification, the pile is assumed to be driven in a rock sea bottom and its top drift limit state is considered. To establish the reasonableness of this representation, the drift at the pile top is estimated using the FE representation shown in Fig. 7 and the analytical solution for elastic beams subjected to external loads. The difference in the drift calculation is found to be less than 1%.

(i) The reliability estimation strategy

The first step in implementing the proposed method is to generate an explicit expression for the implicit response surfaces (RS). Initially, a first-order RS is considered using the FE model shown in Fig. 6. Then, a preliminary reliability analysis is carried out. From the sensitivity analysis, it is observed that, E_s has a very low sensitivity index, about 0.001. It is considered as a deterministic variable at its mean value. It reduces the total number of random variables to four. The results are summarized under first-order polynomial in Table 4.

(ii) Reference value

In order to obtain a reference value to compare with, the explicit limit state is expressed as follows:

$$g(\mathbf{x}) = \delta_{allow} - y_{max}(\mathbf{x}) = 30 - HL_o^3 / (3E_s I) \quad (12)$$

where $y_{max}(\mathbf{x})$ is the calculated top drift of the pile, δ_{allow} is the allowable drift assumed to be 30 cm, I is the moment of inertia

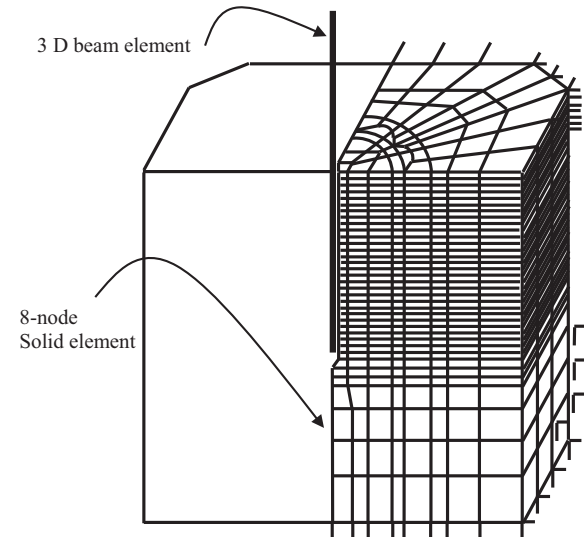
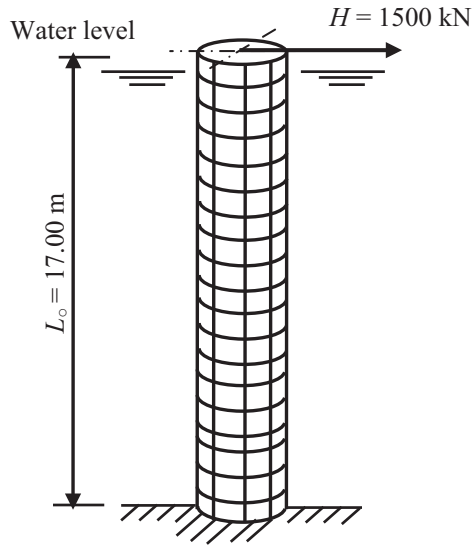


Figure 6 FEM Representation of steel pipe-soil systems.

Table 3 Statistical characterization of random variables - Example 2.

	Random variables	Symbol	Distribution	Nominal	Mean	Bias	COV
1	Lateral load	H	EV-I	1500 kN	1170 kN	0.78	0.37
2	Pile radius	r	LN	0.95 m	0.95 m	1.00	0.10
3	Thickness	t	LN	2.8 cm	2.8 cm	1.00	0.05
4	Cantilever length	L_o	N	17.50 m	17.50 m	1.00	0.05
5	Steel elastic modulus	E_s	LN	2.01×10^8 kN/m ²	2.01×10^8 kN/m ²	1.00	0.06

**Figure 7** Mooring dolphin embedded in rock sea bed.

of the pile and it is calculated as $I = \pi/64[(2r)^4 - (2r - 2t)^4]$, and all the other variables were defined earlier.

Considering all 5 random variables with statistical properties given in Table 3 and using MCS, the probability of failure and the corresponding reliability index are found to be $P_f\text{-MCS} = 5.69 \times 10^{-2}$ and $\beta\text{-MCS} = 1.581$, respectively. When SORM is used, the corresponding information

is almost identical. The results are summarized in Table 4. Considering very low sensitivity index, when E_s is assumed to be a deterministic variable at its mean value, the reliability index and the probability of failure are again estimated using SORM and MCS. As expected, the results shown in Table 4 remain almost the same. Similarities of the results obtained by SORM and MCS confirm the accuracy of the formulation and the reference value for the reliability index can be considered as of the order of 1.58. The task is to show that the reliability indexes obtained by the several RSM-based schemes are also very similar, verifying the proposed concept.

(iii) Response surface method

Using Scheme 0 (SD with second-order polynomial without cross terms), the RS for the top lateral deflection of the pile is found to be

$$g(\mathbf{X}) = \delta_{allow} - \hat{g}(\mathbf{X}) = 30 - [344.66 + 0.18 \times H - 558.47 \times r - 5.02 \times L_o - 3096.55 \times t - 1.0 \times 10^{-15} \times H^2 + 260.5 \times r^2 + 0.28 \times L_o^2 + 37205.62 \times t^2] \quad (13)$$

Similar expressions for Schemes 1, M1, and 2 are generated, but are not shown here. The probabilities of failure using Schemes 0, 1, M1 (3 ways), and 2 are estimated using the proposed concept. For Scheme M1, cross terms are used in 3 different ways. Initially, 3 cross terms of the most significant

Table 4 Results of reliability analysis (RS and Explicit functions-based solutions) – example 2.

		Variables sensitivities, $\alpha(X_i)$					SORM		FORM		No. of FE calls	
		H	r	L_o	t	E_s	β	P_f	β	P_f		
<i>(i) Explicit limit state</i>												
1	Monte Carlo – 5 variables						1.581	5.69×10^{-2}	1.581	5.69×10^{-2}	10^5	
	Monte Carlo – 4 variables						1.581	5.69×10^{-2}	1.581	5.69×10^{-2}	10^5	
2	SORM 5- variables	−0.725	−0.614	−0.296	0.96	0.000	1.584	5.66×10^{-2}	1.580	5.70×10^{-2}	1	
	SORM 4 – variables	−0.725	−0.614	−0.296	0.96	–	1.584	5.66×10^{-2}	1.580	5.70×10^{-2}	1	
<i>(ii) Response surface</i>												
3	First order polynomial	−0.886	0.407	−0.211	0.067	0.000	1.936	2.64×10^{-2}	1.953	2.54×10^{-2}	11	
4	Scheme 0	−0.705	−0.640	−0.292	0.095	–	1.415	7.85×10^{-2}	1.580	5.71×10^{-2}	9	
5	Scheme M1–1	H	−0.712	0.633	−0.289	0.094	–	1.541	6.17×10^{-2}	1.580	5.71×10^{-2}	12
	Scheme M1–2	H & r	−0.712	0.632	−0.291	0.095	–	1.560	5.94×10^{-2}	1.580	5.71×10^{-2}	14
	Scheme M1–3	H, r & L	−0.712	0.632	−0.291	0.095	–	1.560	5.94×10^{-2}	1.580	5.71×10^{-2}	15
6	Scheme 1		−0.712	0.632	−0.291	0.095	–	1.560	5.94×10^{-2}	1.580	5.71×10^{-2}	15
7	Scheme 2		−0.715	0.628	−0.291	0.095	–	1.586	5.64×10^{-2}	1.574	5.77×10^{-2}	25

variable H (Scheme M1-1), followed by 2 cross terms of r (Scheme M1-2), and finally 1 cross term of L (Scheme M1-3, same as Scheme 1) are added. The results are summarized in Table 4.

Several important observations can be made from the results. As expected, Scheme 0 is the least accurate but the most efficient; it requires only 9 FE analyses. The probabilities of failure using Schemes 1 and 2 are slightly different, but they require 15 and 25, FE analyses, respectively. When three modified Schemes are used, they required 12, 14 and 15 FE analyses, respectively. In all cases, the reliability indexes are found to be very similar and very close to the reference value of 1.58. On the other hand, the results using FORM are listed in Table 4. Generally, SORM is more realistic than FORM. However, there are many other factors that affect the results such as; the shape of the limit state and the value of the arbitrary factor, h_i .

Both examples confirm that instead of using thousands of Monte Carlo simulations, similar results can be obtained by using only tens of deterministic evaluations at very intelligently selected points using the proposed concept. The proposed concept is viable.

6. Conclusions

A general purpose reliability analysis method for low probability events for large complicated real nonlinear structures for engineering applications is presented. In this approach, only tens instead of hundreds or thousands of deterministic evaluations at intelligently selected points are used to extract the required reliability information. It is a significant improvement over other methods currently available or being developed in other disciplines. For this class of problems, the basic Monte Carlo simulation cannot be used. The proposed method provides an alternative to the MCS method. Several areas of response surface method including advanced factorial design concepts are improved or developed and presented. The method is elaborated with the help of two informative examples. The reliability of a steel frame excited by the El Centro Earthquake of 1940 is first estimated. To improve its lateral stiffness, reinforced concrete shear walls are added and the reliability of the dual system is then estimated. In the second example, the reliability of offshore mooring dolphin structures is estimated. The proposed method is robust, efficient, and accurate. It can be used to estimate reliability of large complicated engineering systems.

Appendix A. Methods of reliability analyses [34]

A.1. Definitions

Reliability: The reliability is defined as the probability of safety or the complement of the probability of failure. Sometimes reliability and safety are used as synonyms.

Safety margin:

$$Z = R - S \quad (A.1)$$

where

R , S : resistance and stress resultant.

Z is a point of failure with a unique/invariant value.

Probability of failure (P_f):

If the allowable resistance is R and the applied stress is S with probability density function f_R and f_S , respectively, then the probability of failure is the amount of overlap of the probability density functions f_R and f_S (in this work, f_R , f_S and the amount of overlap are assumed to be time independent). In another form, let f_R and f_S be two marginal density functions as shown in Fig. A1 where F_R is the resistance cumulative function and both R and S are independent then,

$$P_f = p(R - S \leq 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{S \geq R} f_R(r) f_S(s) dr ds = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx \quad (A.2)$$

In a more general form, the random variables affecting the response are grouped in a vector called the vector of basic random variables X .

$$P_f = P[G(X) \leq 0] = \int \cdots \int_{G(X) \leq 0} f_X(X) dX \quad (A.3)$$

where

$f_X(X)$: joint probability density function of n basic variables X .

$G(x)$: the limit state function.

Calculation of Probability of failure P_f

As previously mentioned, the probability of failure can be defined as

$$P_f = \int_{G(X) < 0} \cdots \int f_X(X) dX \quad (A.4)$$

The above multidimensional probability convolution integral is rather tedious. However, P_f may be directly calculated by numerical methods for simple cases otherwise two main categories of methods may be used. They are denoted as the simulation methods and the fast integration methods.

A.2. Simulation methods

The Monte Carlo Simulation (MCS) technique involves sampling process randomly to simulate a large number of

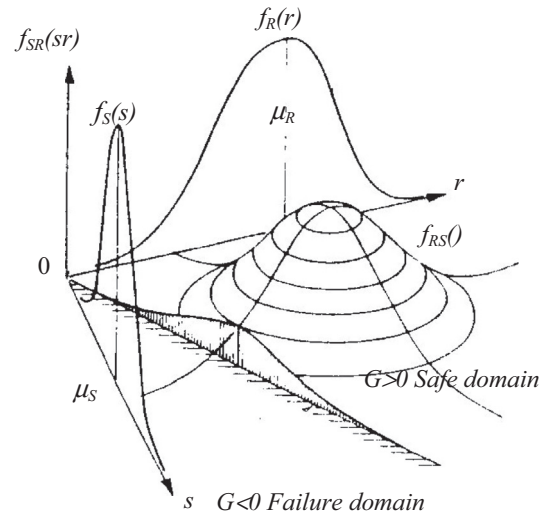


Figure A1 Domain of failure probability.

experiments and observe the result. If the number of sampling N with n failure states, then

$$P_f \approx n(G \leq 0)/N \quad (\text{A.5})$$

$$P_f \approx \hat{P}_f = \frac{1}{N} \sum_{i=1}^N I[G(\hat{X}_i) \leq 0] \quad (\text{A.6})$$

where $I[G(\hat{X}_i) \leq 0]$ is an indicator function of $G(x)$ equal one if X lies in the failure domain and zero otherwise. N depends on the required accuracy.

The sampling is obtained randomly using tables of random numbers or using a pseudo random number generator which uses the local time as a seed value to avoid any reproductively. However, using the tables is very slow and using the pseudo random number generator may be criticized as it is no longer random as the sequence of the numbers is determined. So, it may be called quasi MCS.

The previous MCS technique is the simplest form and may be called “direct sampling” or “Crude Monte Carlo”. Other modified methods such as variance reduction, importance sampling, and adaptive Monte Carlo are found in Melchers [34].

A.3. Fast integration methods

These methods are based on the simplicity of finding the integral in the standardized space. So, all basic variables X_i are transformed to uncorrelated standardized distributed variables U_i . Also, the limit state function $G(x)$ is transformed to $G(u)$ [35]. Hence, P_f may be estimated by one of the following methods.

A.3.1. First Order Reliability Method (FORM)

Hasfor and Lind [36] have initially proposed this method in 1974. In 1978 Rackwitz and Fiessler [37] have put the solution in an algorithmic form. In the basic FORM [38], the limit state $G(u)$ in u -space is approximated by its hyperplane in the $G(u)$ at a point (u^*) closest to the origin. By this way the multidimensional integral problem is converted to an optimization problem for finding the shortest distance between the origin and the hyperplane which is called the reliability index β

$$P_f \approx \Phi(-\beta) \quad (\text{A.7})$$

$$\beta = \|U^*\| \quad (\text{A.8})$$

where $u^* = \min \|u\|$ for $\{u: g(u) \leq 0\}$.

The optimization problem requires that the distribution of X and $G(u)$ should be differentiable. This method yields sufficiently accurate probability of failure estimation for most engineering proposes, COMREL [39]. Through this method, the probability of failure for concave and convex limit state function is the same as that of linear limit state function provided that they have the same check point as shown in Fig. A2.

A.3.2. Second Order Reliability Method (SORM)

Obviously, the linear approximation of the true failure surface in FORM appears to be rather crude. Breitung in 1984 [40] has given a sound theoretical basis SORM using a quadratic approximation of the failure surface by use of asymptotic consideration which has been modified in COMREL [39] according to the following formula:

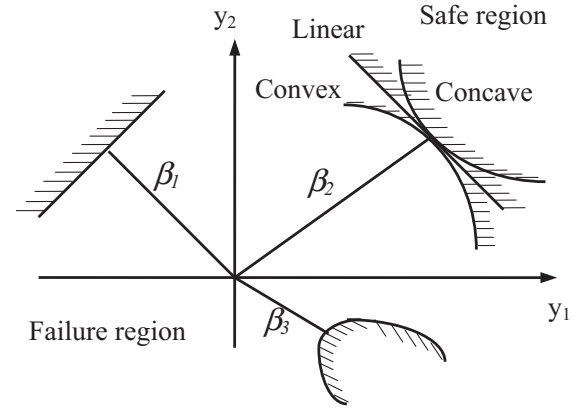


Figure A2 Different limit state and common checking point.

$$P_f \approx \Phi(-\beta) \prod_{i=1}^{n-1} \left(1 - \frac{\varphi(-\beta)}{\phi(-\beta)} \kappa_i \right)^{\frac{1}{2}} \quad (\text{A.9})$$

where $\beta = \|U^*\|$ in which u^* is found from $u^* = \min \|u\|$ for $\{u: g(u) \leq 0\}$.

The difference between linear and quadratic approximation of nonlinear surface increases with problem dimensions and safety index. It drastically depends on the curvature in the checking point [41]. SORM appears to be more accurate than FORM. However, the checking point and curvature is not sufficiently representative for the entire shape of the failure surface. Besides, the limit state surface must be continuous and twice differentiable.

Appendix B. Design of experiments (FE simulation)

The RSM in statistics has two major considerations: design or planning and estimation. Design is considered with how best to locate the points in the statistical variables space at which experiments will be run so that the fitting of the response surface to the true response will satisfy certain criteria. Estimation treats the question of how to use the y 's and x 's to calculate the coefficients appearing in the specification of g .

The choice of sets of basic variables for deterministic design/experiments has a pronounced effect on the accuracy of approximation. These sets can be fixed according to certain plan. The feasible domain for n -basic can be considered as a part of k -dimensional space. The origin of the coordinates is the point with mean values of the variables. A hyperparallelepiped with the center at the origin of coordinates and with faces perpendicular to coordinate axis is built. The hyperparallelepiped has 2^k vertices and $2k$ faces. For 3 variables, see Fig. B1(a).

The design at the vertices of hyperparallelepiped is called 2-level factorial plan [42,43]. 3^k factorial plan can be used with central value equal zero, see Fig. B1(a and b), respectively.

With large k , the full factorial plan requires not only more effort and cost but also lengthy computations. So, saturated plan without edge points or with edge point and the central composite design/plan (CCD) can be used, as shown in Fig. B1(c-e). The saturated design consists of a central point and $2k$ axial points. Edge points may be added (saturated design with edge points). This plan consists of 2^k factorial design, a center point and two axial points on the axis of each random variable at a distance c from the center point. The

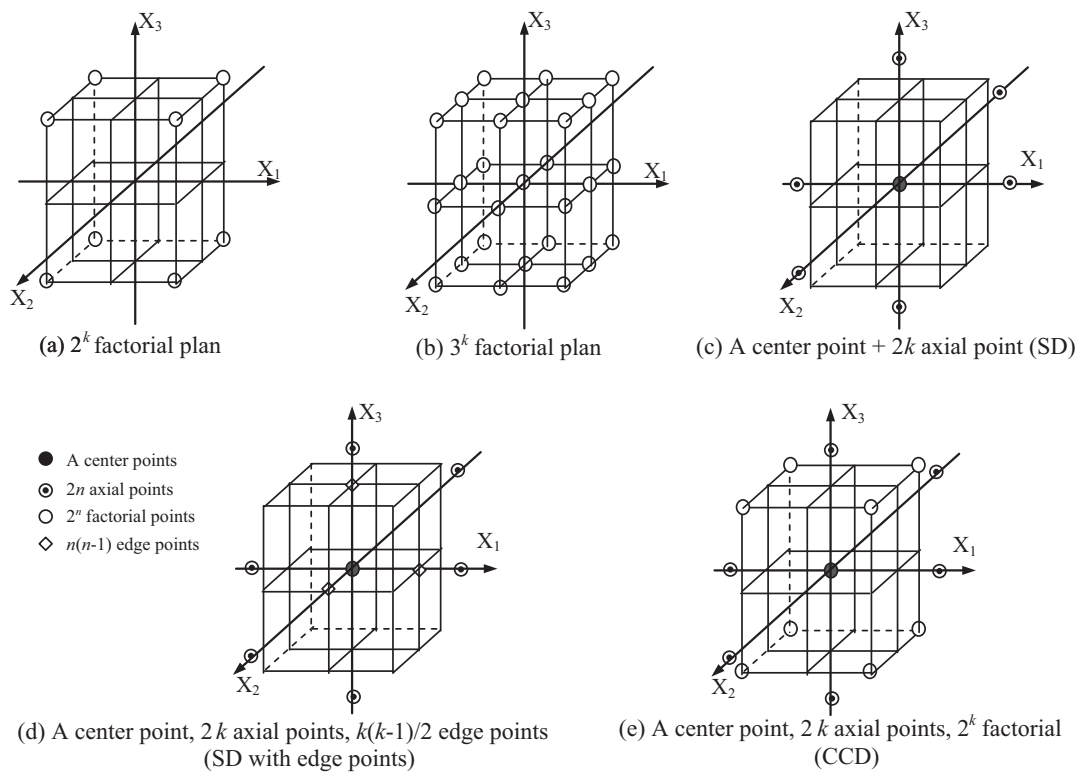


Figure B1 Different models of experimental sampling.

designer can choose the value of c which is frequently chosen $c \geq 1$.

However, the exponential increase in the number of function calls with the number of basic variables k , leads to unacceptable high computational efforts for complex structural systems. Therefore Bucher and Bourgund [9] have indicated that the response surface need not to be exact in the entire space but only the sign of $g(x)$ near the actual design point, i.e. the region which contributes most to the total failure probability. It is found that, this representation makes the final reliability analysis very close to the exact results. The general form of the approximated function in this method is quadratic polynomial:

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^k b_i X_i + \sum_{i=1}^k b_{ii} X_i^2 \quad (\text{B.1})$$

where X_i ($i = 1, 2, \dots, k$) is the i th random variable, k is the number of random variables in the formulation, ($j = 1, 2, \dots, k$) and b_0 , b_i , b_{ii} , and b_{ij} are unknown coefficients to be determined.

The first point is chosen to be the mean values of the basic variables; the rest $2k$ points are chosen to be

$$X_i = X_i^C \pm h_i \sigma_{X_i} \quad i = 1, 2, \dots, k \quad (\text{B.2})$$

where X_i^C and σ_{X_i} are the coordinates of the center point and the standard deviation of a random variable X_i , respectively, h_i is an arbitrary factor that defines the sampling/experimental region. Its value can be assumed from 1 to 3.

This procedure needs only, $2k + 1$, points as shown in Fig. B1(c). Fig. B1, sums up some famous plans in literature. Table B1 compares the required number of function calls for each plan.

Table B1 The required number of FE calls to different types of plan.

No.	Method of plan	No. of calls to G (X)	Figure
1	2^k factorial plan	2^k	a
2	3^k factorial plan	3^k	b
3	Saturated design (SD)	$1 + 2k$	c
4	Saturated design with edge point	$k(k-1)/2 + 2k + 1$	d
5	Central composite design (CCD)	$2^k + 2k + 1$	e

To be efficient, design of experiments should be concerned with how best to locate the points in the vicinity of failure point; a point which is not at hand. However, this point can be reached iteratively by first making the numerical experiments around initial point which may be taken as the mean. Then the procedure is repeated in an iterative strategy until achieving acceptable level of accuracy.

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